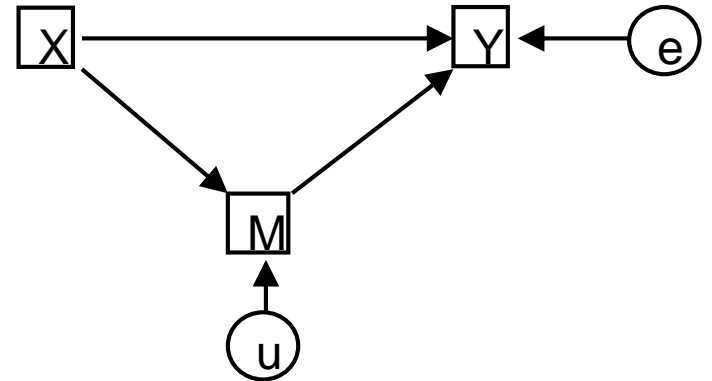


# TCSM Mediation



# OUTLINE

- Relationships among 3 variables
- investigating mediation
- testing the indirect (or mediated) effect



Sharing childcare matters, new [@ESR\\_news](#) paper by [@renske\\_keizer](#), [@cjvanlissa](#) & colleagues.  
Ungated paper!



**ESR** @ESR\_news · 1h

New paper by Keizer and colleagues shows that parents' equally sharing childcare responsibilities functions as an underlying mechanism for social class disparities in children's cognitive development [doi.org/10.1093/esr/jc...](https://doi.org/10.1093/esr/jc...)

*European Sociological Review*, 2019, 1–15

doi: 10.1093/esr/jcz046

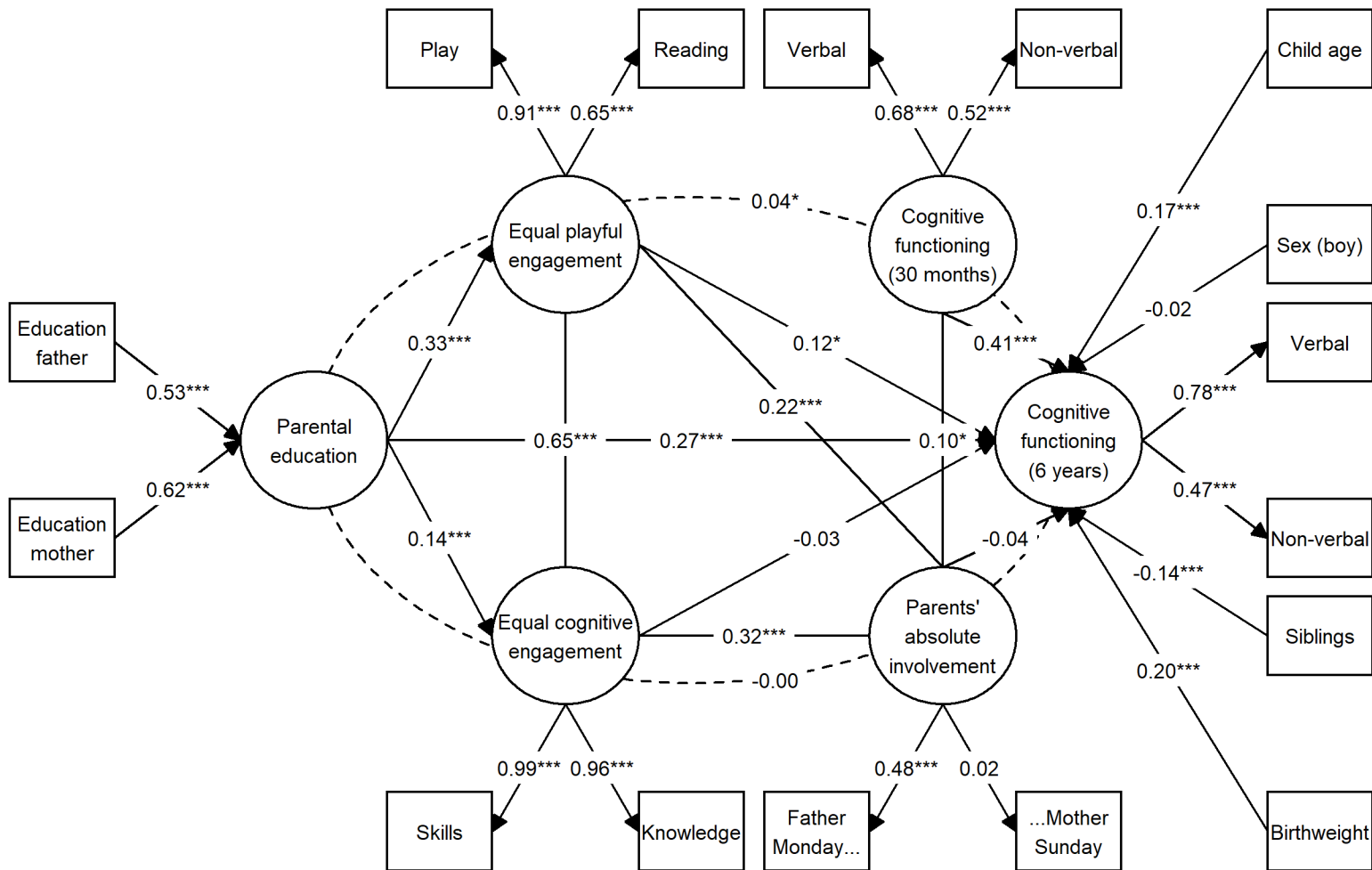
Original Article

OXFORD

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## **The Influence of Fathers and Mothers Equally Sharing Childcare Responsibilities on Children's Cognitive Development from Early Childhood to School Age: An Overlooked Mechanism in the Intergenerational Transmission of (Dis)Advantages?**

Renske Keizer<sup>1,\*</sup>, Caspar J. van Lissa<sup>2</sup>, Henning Tiemeier<sup>3,4</sup> and Nicole Lucassen<sup>5</sup>



# Mediation

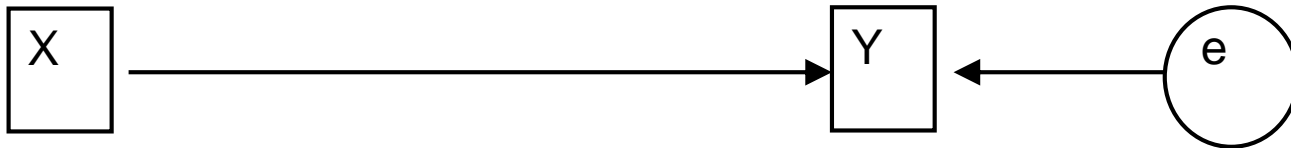
Snooping on your teenager leads to more parent-child conflict

WHY?

Because snooping interferes with teenagers' **autonomy needs**

# Mediation

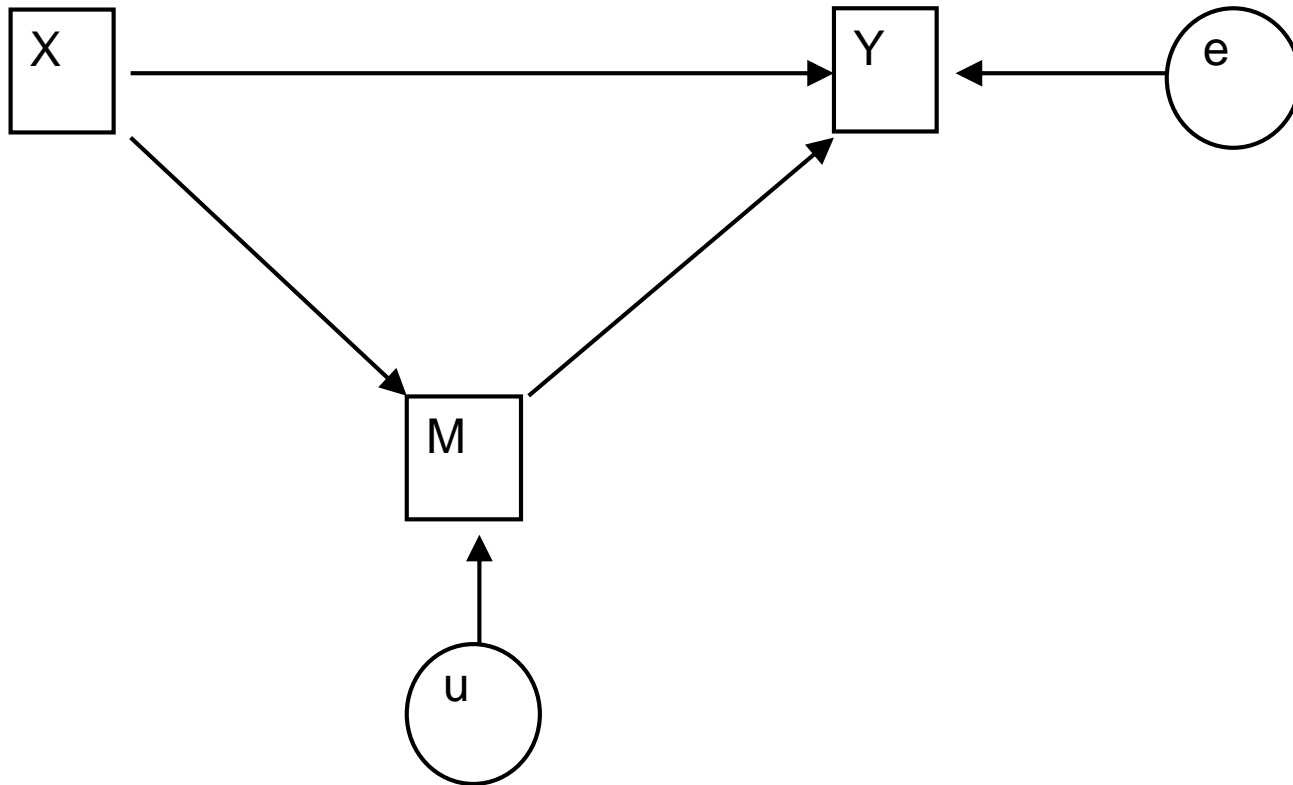
Snooping leads to more parent-child conflict



u

# Mediation

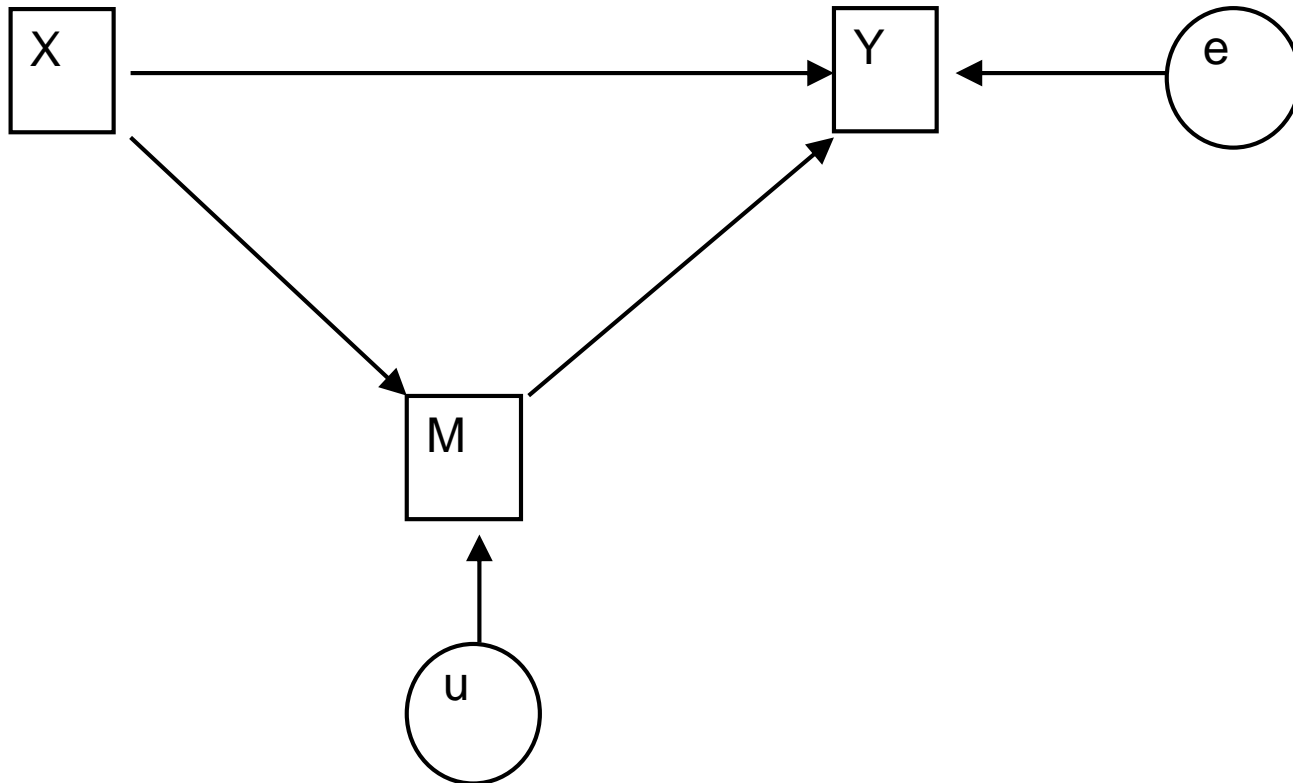
...because snooping frustrates autonomy needs!



# Mediation

The effect of X on Y is (partially) **mediated** by M

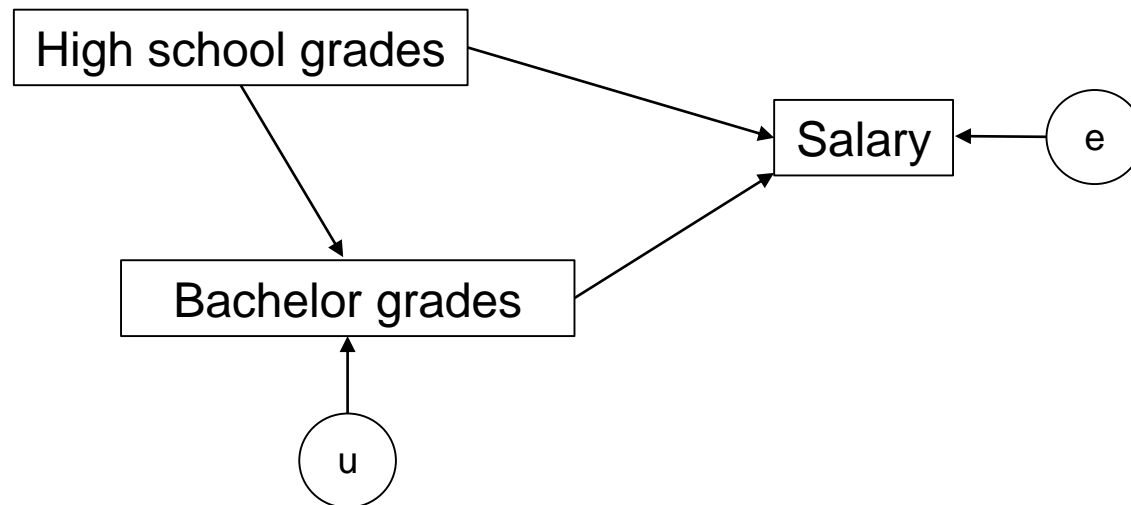
Mediated: Explained by





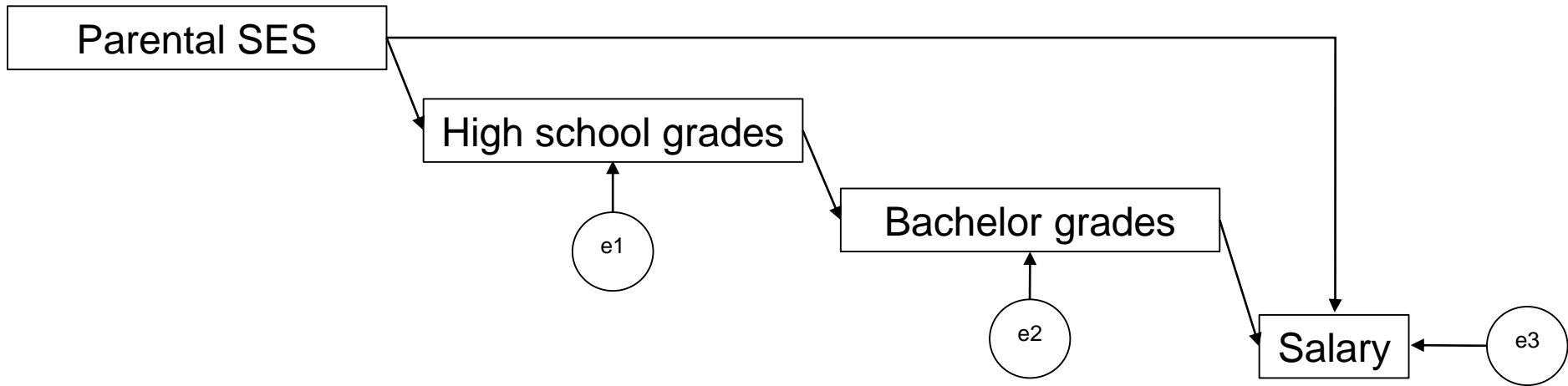
# Mediation

$X$  influences  $Y$  through a third variable: Mediator  $M$ .



# Mediation

$X$  influences  $Y$  through a third variable: Mediator  $M$ .



Effects in a mediation model

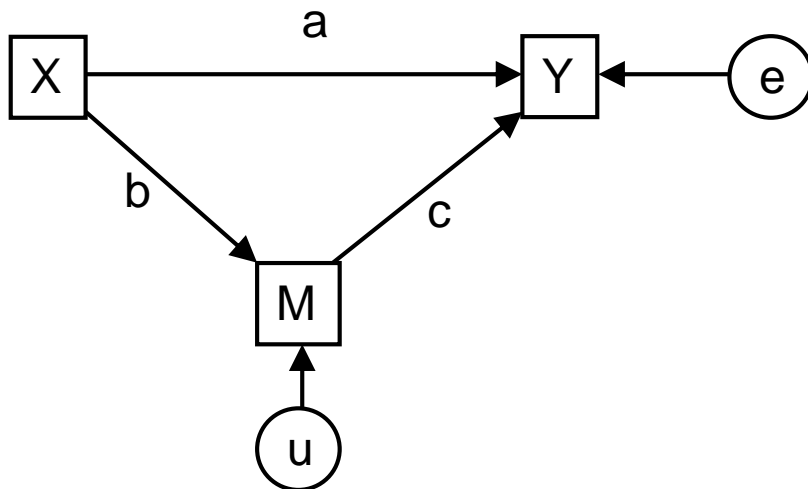
# Direct and indirect effects

The influence of X on Y is (partially) mediated by M, We also say: X has an indirect effect on Y

**Direct effect** of X on Y:  $a$

**Indirect effect** of X on Y:  $b*c$

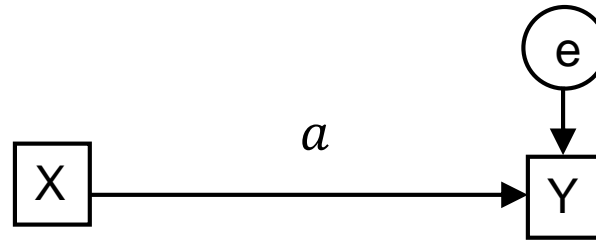
**Total effect** of X on Y:  $a + (b*c)$



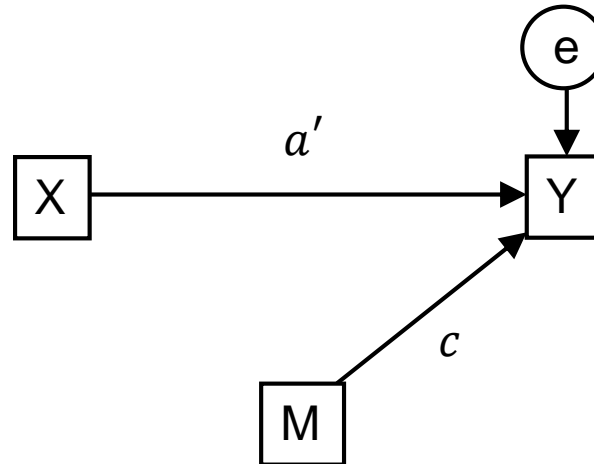
# Determine indirect effect using regression:

## Approach 1:

1) Simple regression



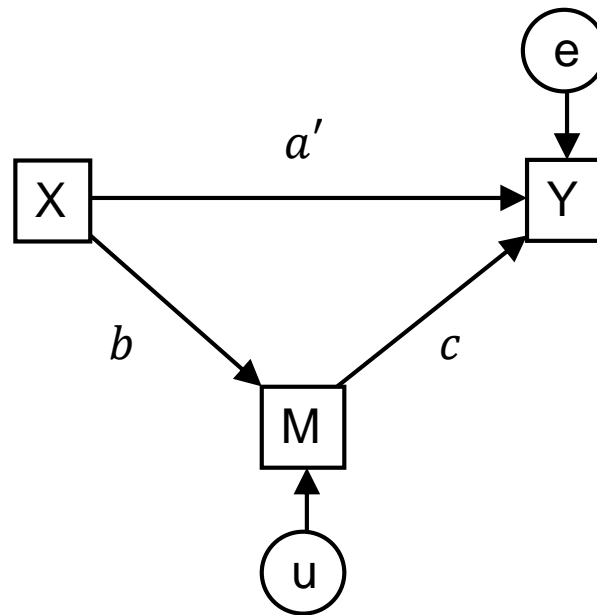
2) Multiple regression



Indirect effect:  $a - a'$

# Determine indirect effect in SEM:

## Approach 2:



Indirect effect:  $b * c$

# OUTLINE

- relationships between 3 variables
- investigating mediation
- testing the indirect (or mediated) effect

# Investigating mediation: old school

**Baron and Kenny** steps (cited 14.000+ times!!!).

Journal of Personality and Social Psychology  
1986, Vol. 51, No. 6, 1173–1182

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0022-3514/86/\$00.75

## The Moderator–Mediator Variable Distinction in Social Psychological Research: Conceptual, Strategic, and Statistical Considerations

Reuben M. Baron and David A. Kenny  
University of Connecticut

In this article, we attempt to distinguish between the properties of moderator and mediator variables at a number of levels. First, we seek to make theorists and researchers aware of the importance of not using the terms *moderator* and *mediator* interchangeably by carefully elaborating, both conceptually and strategically, the many ways in which moderators and mediators differ. We then go beyond this largely pedagogical function and delineate the conceptual and strategic implications of making use of such distinctions with regard to a wide range of phenomena, including control and stress, attitudes, and personality traits. We also provide a specific compendium of analytic procedures appropriate for making the most effective use of the moderator and mediator distinction, both separately and in terms of a broader causal system that includes both moderators and mediators.





# Three steps of B&K

Several regressions, puzzle together the path model:

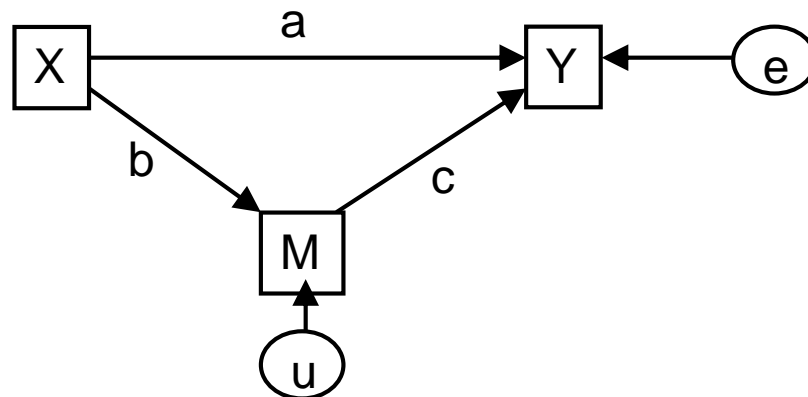
**Step 1:** Is  $X$  a significant predictor of  $Y$ ? ( $a$ )

**Step 2:** Is  $X$  a significant predictor of  $M$ ? ( $b$ )

**Step 3:** In model with both  $M$  and  $X$  as predictors, is  $M$  a significant predictor of  $Y$ ? ( $c$ ).

Did  $a$  decrease?

(or increase?)



# Some problems with B&K

If there is a **suppression effect**

*(the direct and indirect effects **cancel out**), then*

Step 1 would not show a significant effect

Also: Low power when using B&K three steps

# Investigating mediation: new school

## Use **Structural Equation Modeling**

### Advantages:

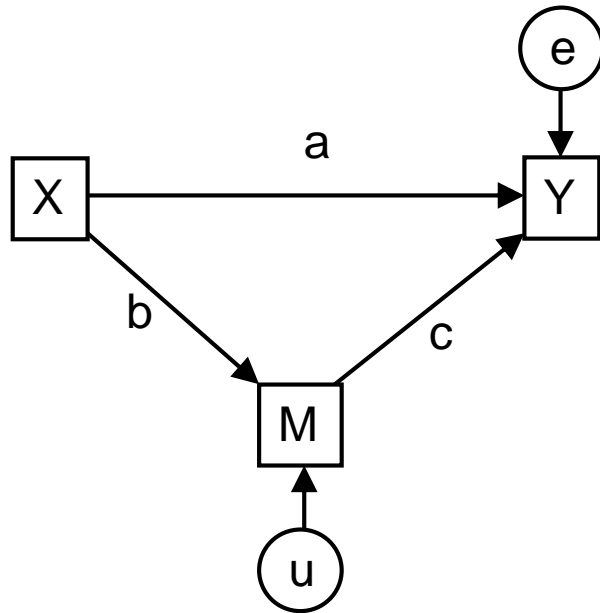
- easier (run 1 model, or 2 nested models)
- Automatically get direct, indirect and total effects; also standardized, and with SEs
- Easily investigate more complicated mediation, e.g.:
  - multiple mediators of one predictor,
  - multiple predictors with one mediator,
  - multiple outcome variables,
  - latent variables

# Mediation in SEM

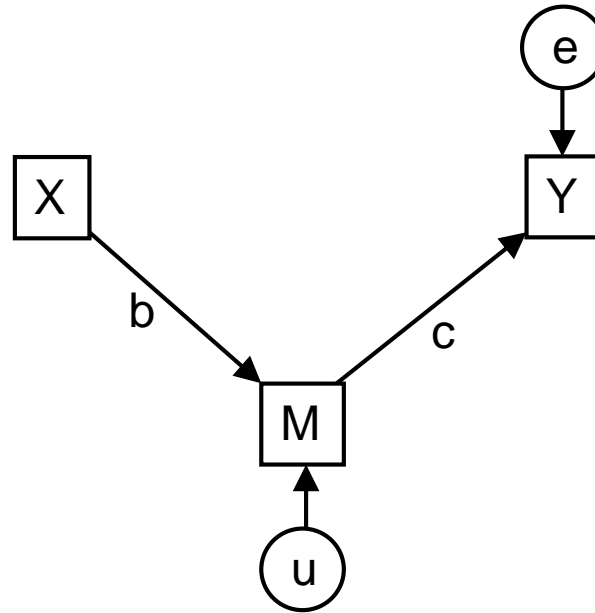
To investigate whether the effect is **fully mediated**, we can do two things:

- 1: Check the **significance** of coefficient  $a$
- 2: Compare **nested models** (which is more complex?)

Model 1



Model 2



# Nested models in SEM

**Nested:** By constraining (to be equal/to be zero) some parameters in model 1, you get model 2.

Compare nested models with a **chi-square diff test**,  
 $\Delta\chi^2$

Both models have a **model-implied vcov matrix**,  $\hat{\Sigma}$

These are compared to the **observed vcov matrix**,  $\mathbf{S}$

$\Delta\chi^2$  is based on comparing the "distance" between  $\mathbf{S}$  and  $\hat{\Sigma}_1$  with the "distance" between  $\mathbf{S}$  and  $\hat{\Sigma}_2$

# Nested models in SEM

- Remember Occam's razor:  
All else being equal, we should prefer simpler models
- Complex models have more "flexibility" to fit data
- Balance necessary complexity and elegant simplicity
- Model 2 has 1 parameter less; does this simplification make the fit significantly worse
- If difference is significant, model 2 isn't supported by the sample covariance matrix

# Nested SEM models: removing path

Is the distance larger?

larger Chi-square

significant result

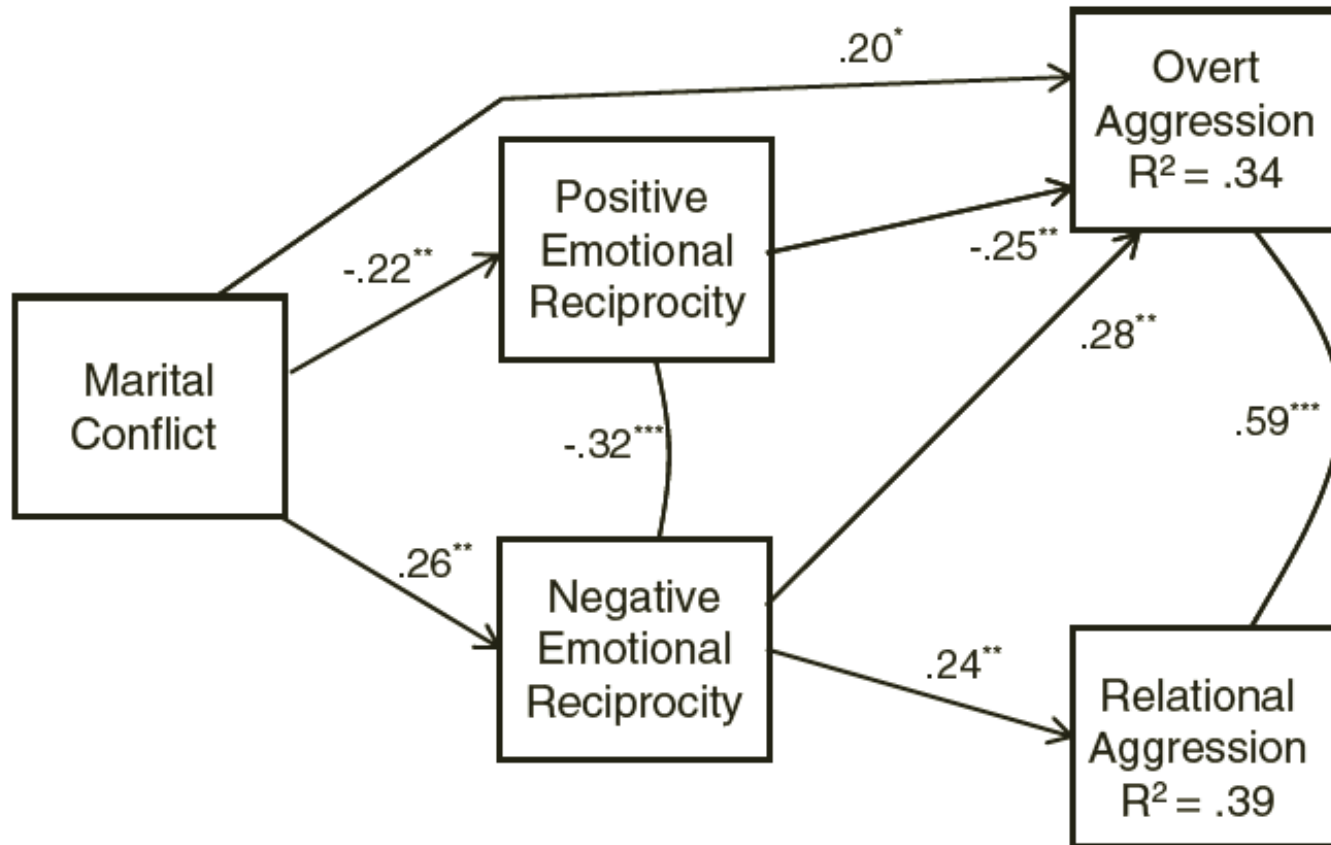
Model 2 is **not** an improvement, Model 1 is better

In contrast, a non-significant result  
means that Model 2 fits equally well, but is simpler:  
choose model 2

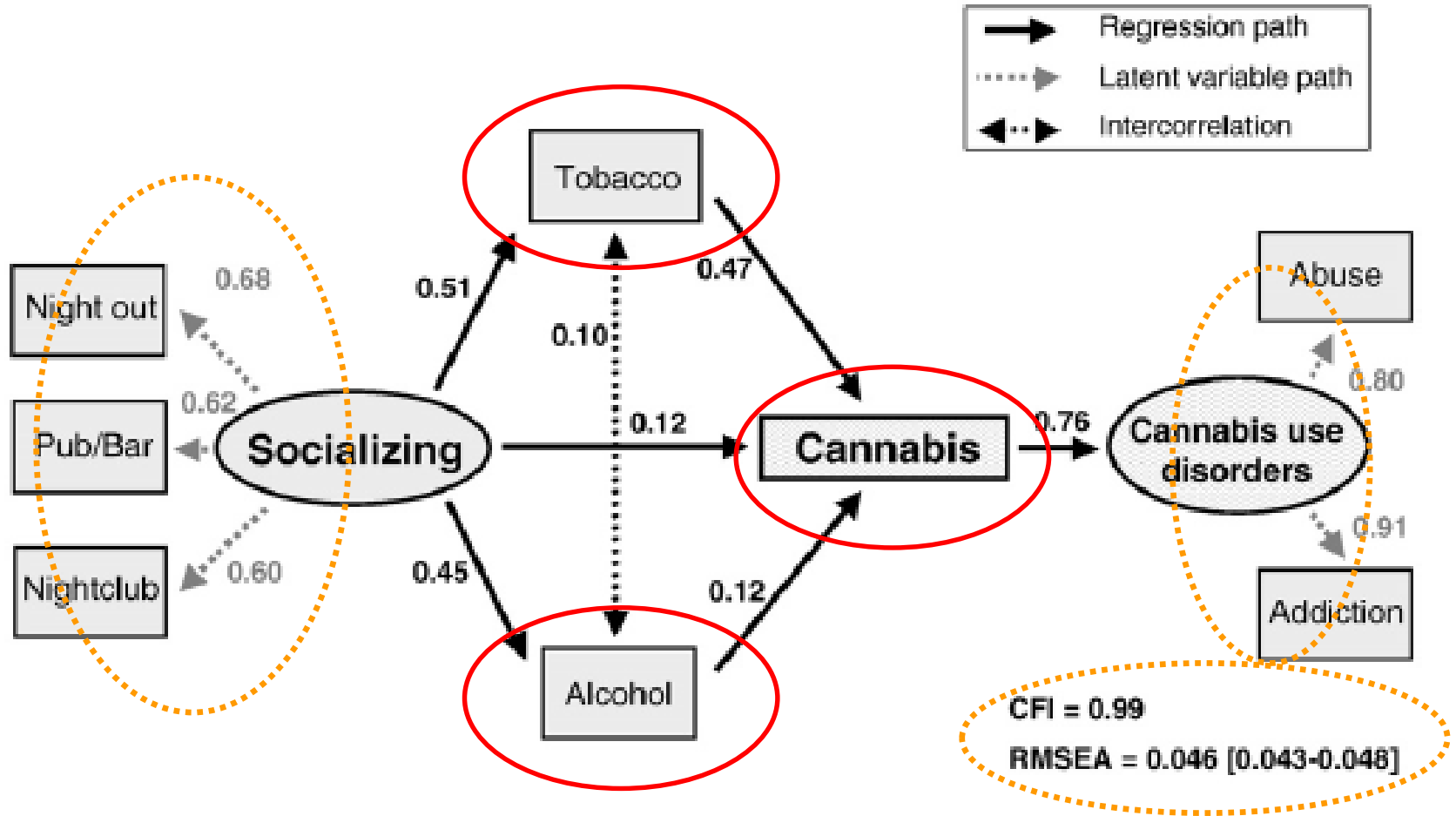
Applied mediation examples



# Aggression in adolescents



# Cannabis use disorders



# Emo. intelligence and life satisfaction

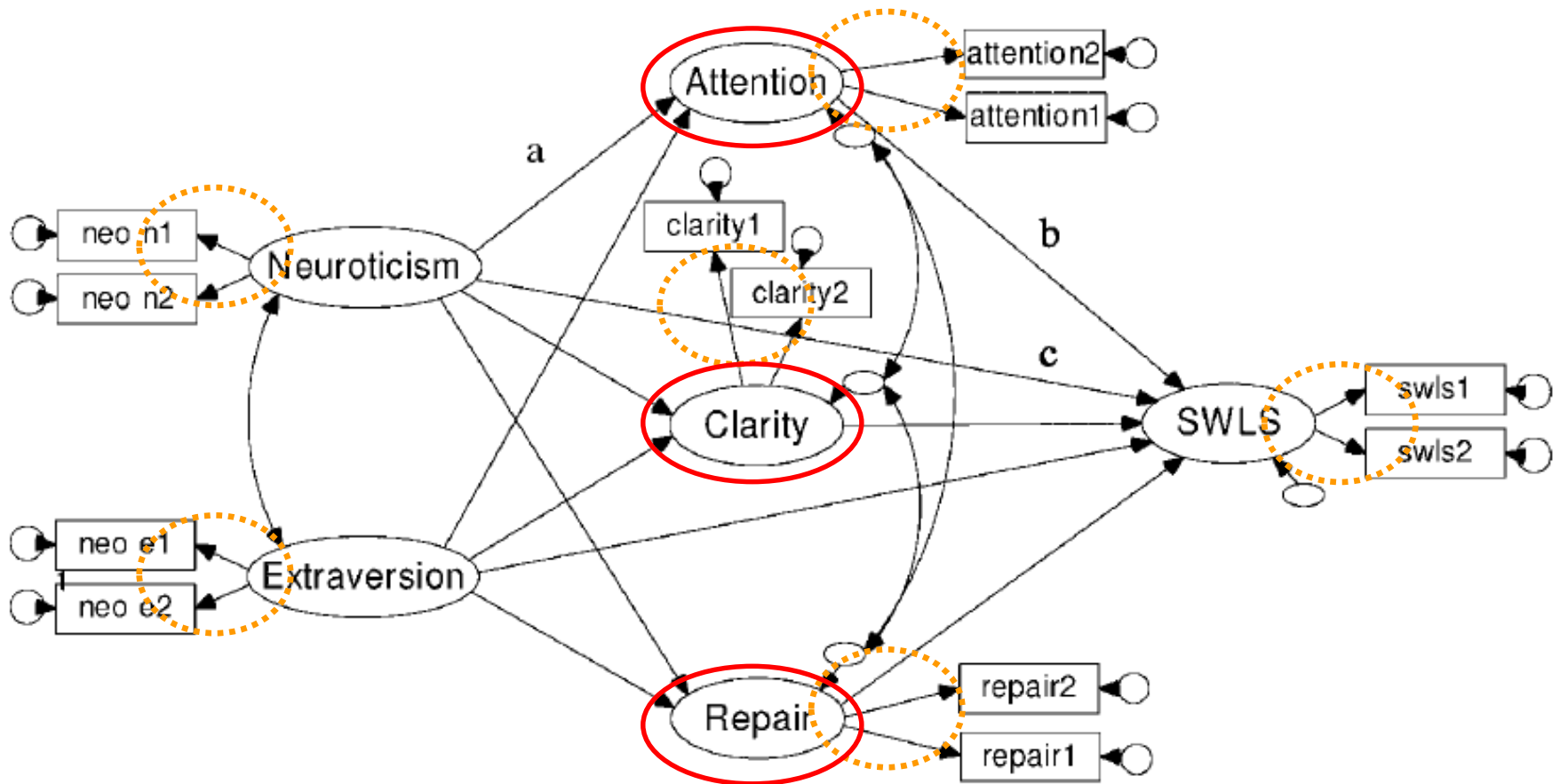


Table 1  
SWLS Items and Factor Loadings

Item	Factor Loadings	Item-Total Correlations
1. In most ways my life is close to my ideal.	.84	.75
2. The conditions of my life are excellent.	.77	.69
3. I am satisfied with my life	.83	.75
4. So far I have gotten the important things I want in life.	.72	.67
5. If I could live my life over, I would change almost nothing.	.61	.57

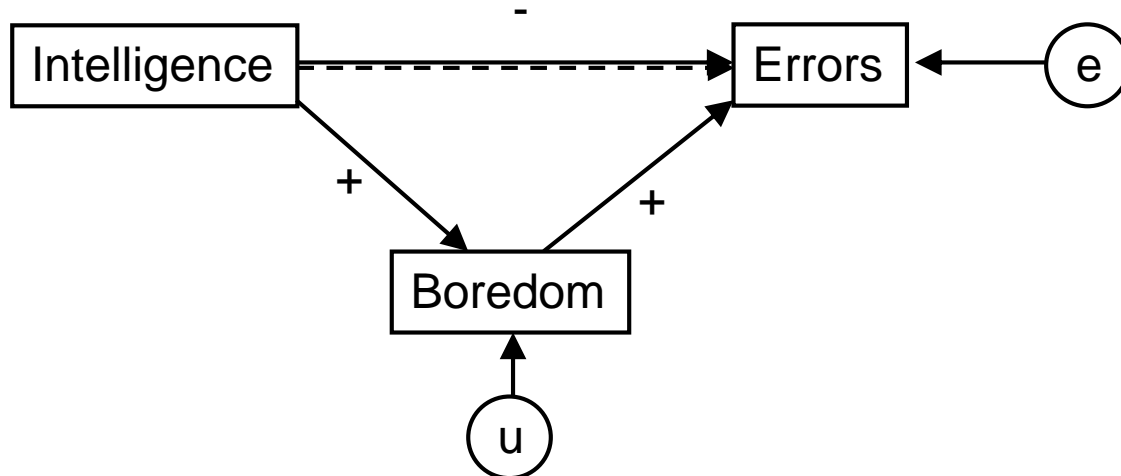
*Note:*  $n = 176$ . SWLS = Satisfaction With Life Scale.

Special cases

# Suppressing mediation

## Suppression:

The direct and the indirect are of **opposite signs**, and (partly) cancel each other out.



Note: In case of suppression, including the mediator will **increase** the predictive ability of X on Y.

# OUTLINE

- relationships between 3 variables
- investigating mediation
- testing the indirect (or mediated) effect
  - Testing a parameter estimate (in general)
  - Testing the indirect effect ( $Ind := b*c$ )
    - Classic: Sobel test - WARNING!
    - Better: Bootstrapping procedure in SEM

# Testing indirect effects

Introducing bootstrapping



# Testing a parameter estimate

Central limit theorem:

The **sampling distribution** for many parameters is (approximately) **normal**.

The sampling distribution is the distribution we would get if we would:

- Take many samples from the same population of the same size
- Estimate the parameter of interest (i.e.,  $\theta$ ) each time

# Testing a parameter estimate

- 1) Estimate the parameter in the sample, e.g.  $T$
- 2) Estimate the SE of the parameter,  $SE_T$
- 3) Derive the sampling distribution **under the null hypothesis** (i.e.,  $\theta_0=0$ ,  $SE = SE_T$ )

In other words: We draw a normal distribution with mean = 0 and sd =  $SE_T$

We then test:

How likely is it to get a value for  $T$  at least as extreme as we observed in our data, IF the null hypothesis were true?

# Distribution of Sample Means

Population mean:

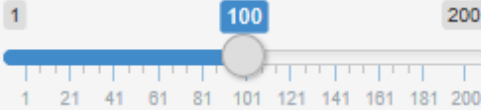
175

Population SD:

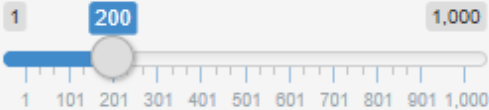
20

Add samples one at a time

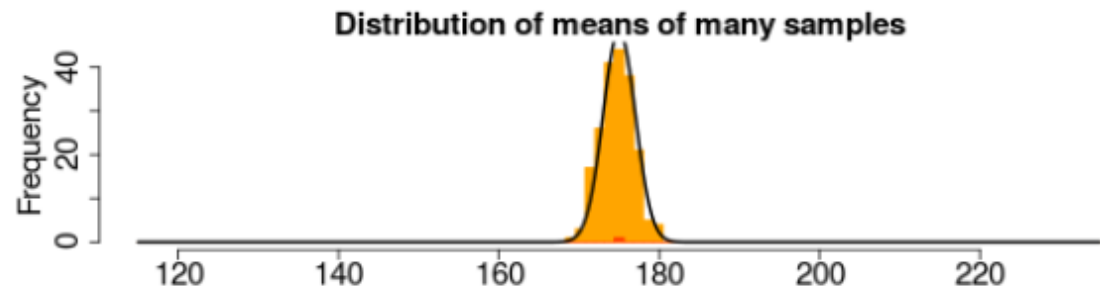
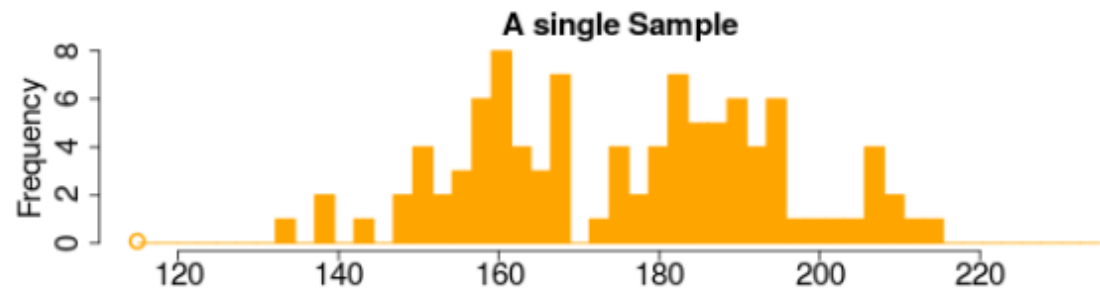
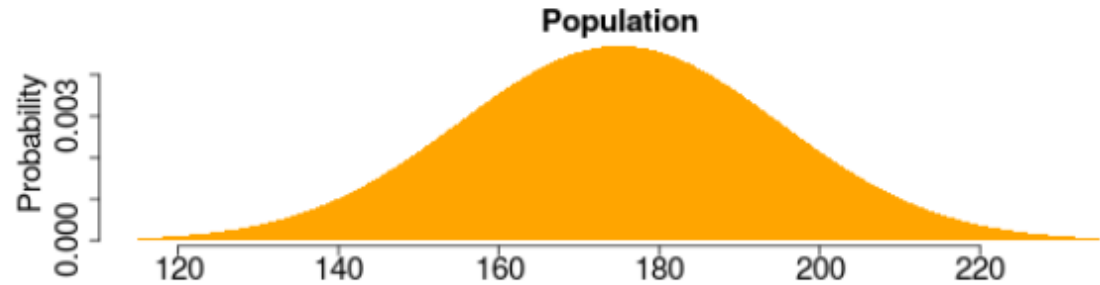
Sample size:



Number of repetitions:



Draw New Sample

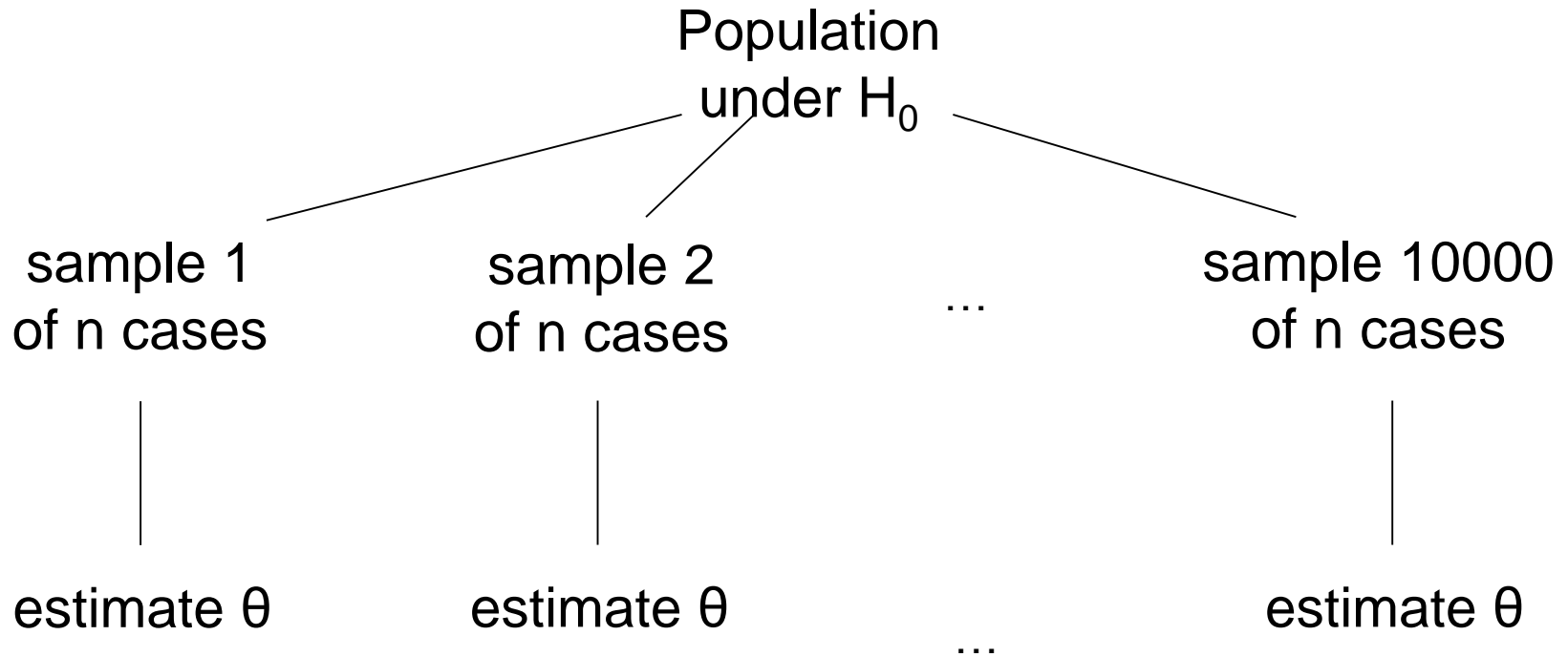


Source: [https://utrecht-university.shinyapps.io/cjvanlissa\\_sampling\\_distribution/](https://utrecht-university.shinyapps.io/cjvanlissa_sampling_distribution/)

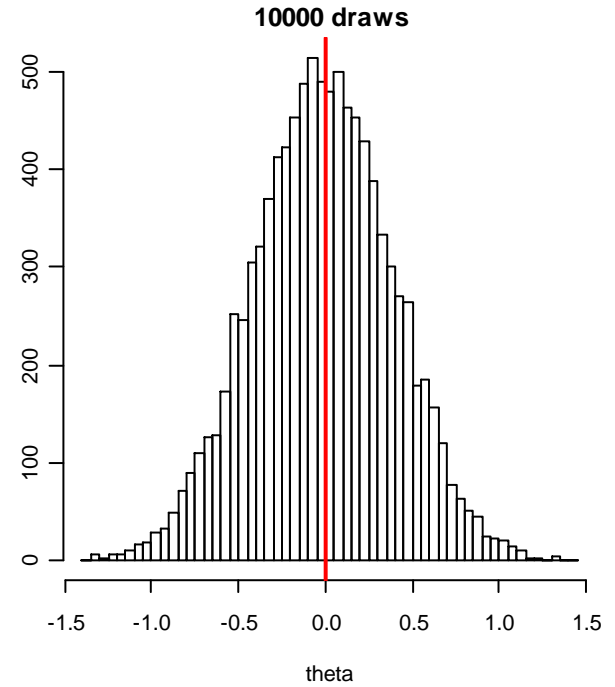
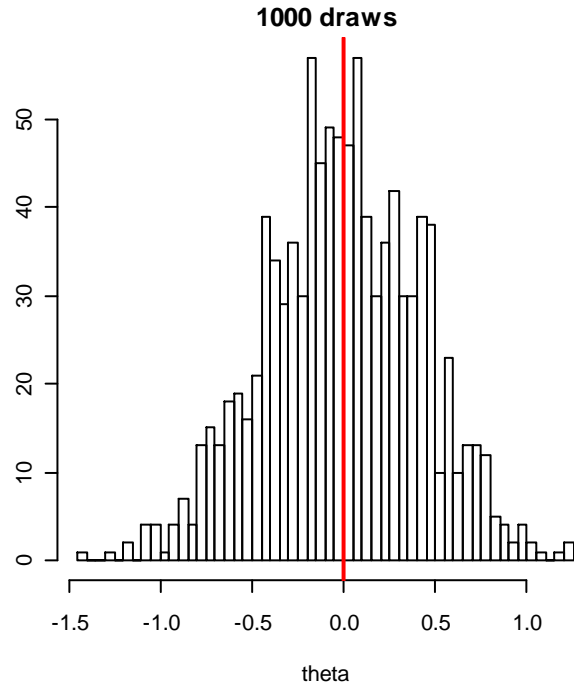
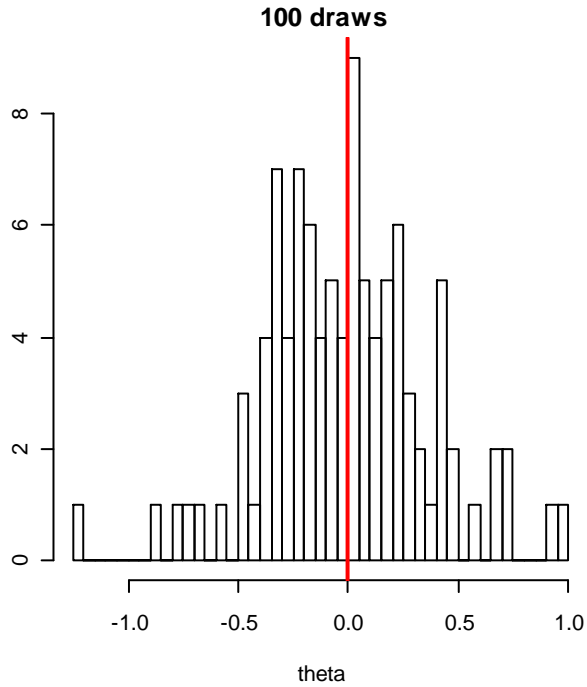
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Edit

# Estimate normally distributed?



# Yes, normally distributed!



Under  $H_0$ :

$$\hat{\theta} \sim N\left(\theta = 0, \sigma_{\hat{\theta}_{(n)}}^2\right)$$

# Standard error

The **standard error** is an estimate of the **standard deviation** of the sampling distribution.

Hence, it can be used to compute a z-statistic and matching p-value (under  $H_0: \theta=0$ ):

$$z = \frac{\hat{\theta}}{SE_{\theta}}$$

Alternatively, one can compute a 95%-confidence interval around the parameter estimate:

$$\hat{\theta} \pm 1.96 * SE_{\theta}$$

# Test indirect effect: Sobel test

The **sampling distribution** for many parameters is (approximately) **normal**.

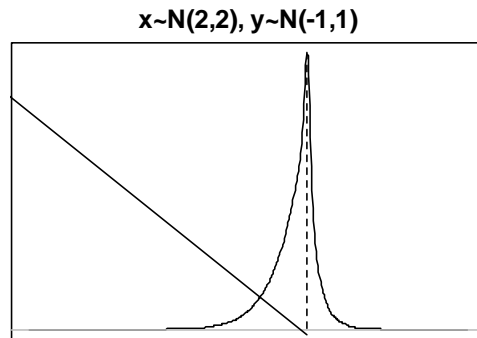
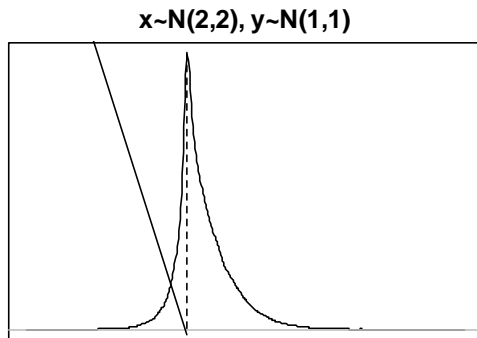
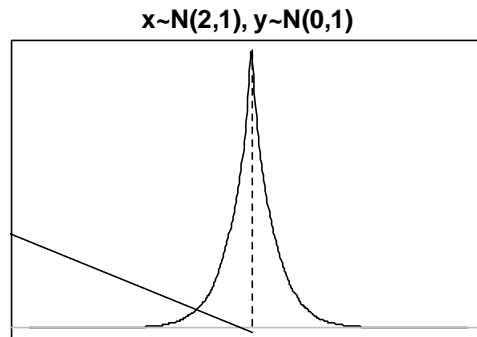
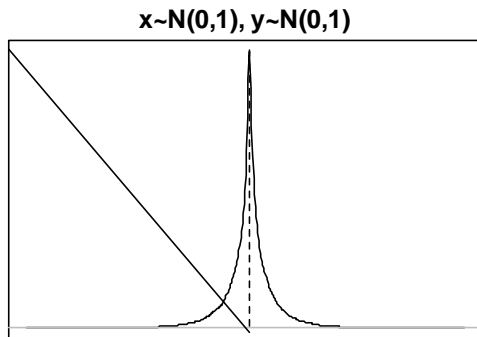
Hence a **z-test** is appropriate.

**Sobel test** for an indirect effect is based on the **assumption** that the sampling distribution of the product of coefficients ( $b*c$ ) is normal.

# Normal product distribution

The **indirect effect** ( $=b*c$ ) is the product of two normally distributed variables.

This does **not** result in a normally distributed quantity!



As a result, the p-value is incorrect (may be too small or too large).



# Solution: bootstrapping

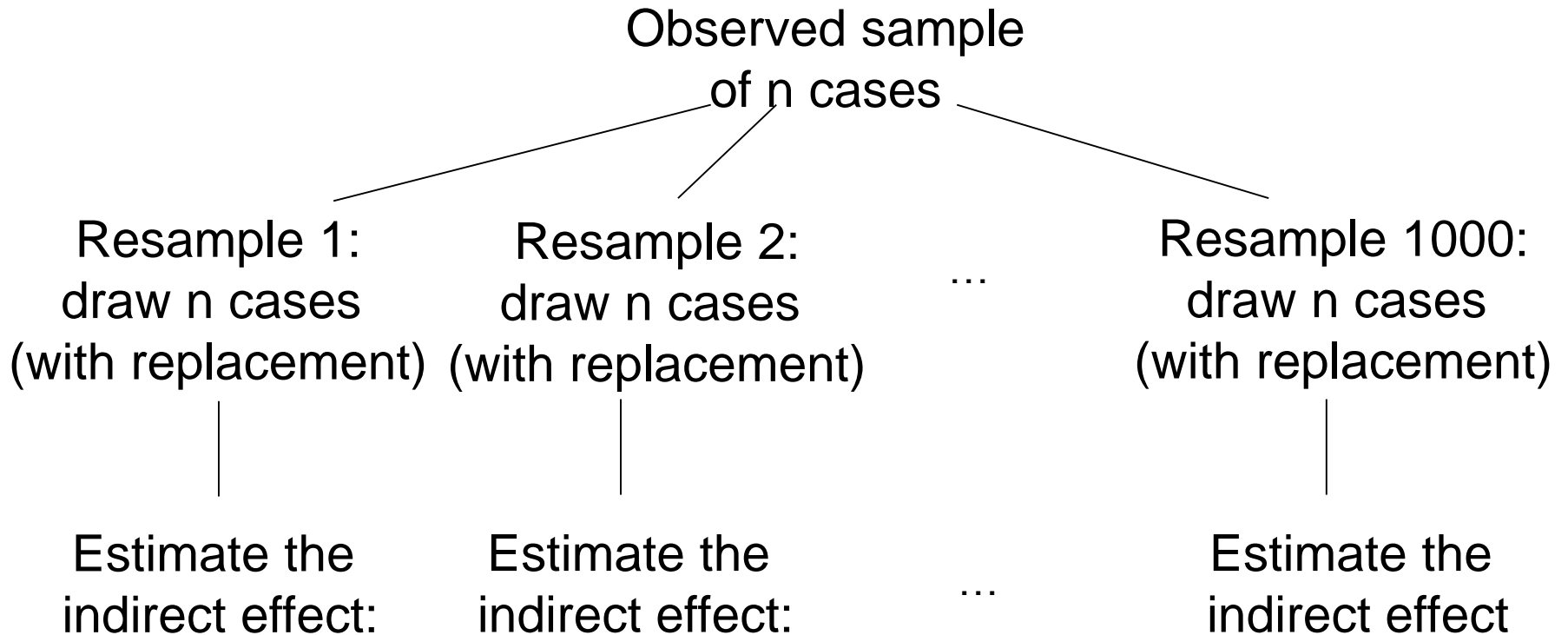
We can **bootstrap** our confidence intervals:



- 1) Re-sample your data (1000x).
- 2) Estimate same model on each bootstrap sample
- 3) Treat the distribution of parameters across bootstrap samples as a sampling distribution

We've "empirically derived" the sampling distribution

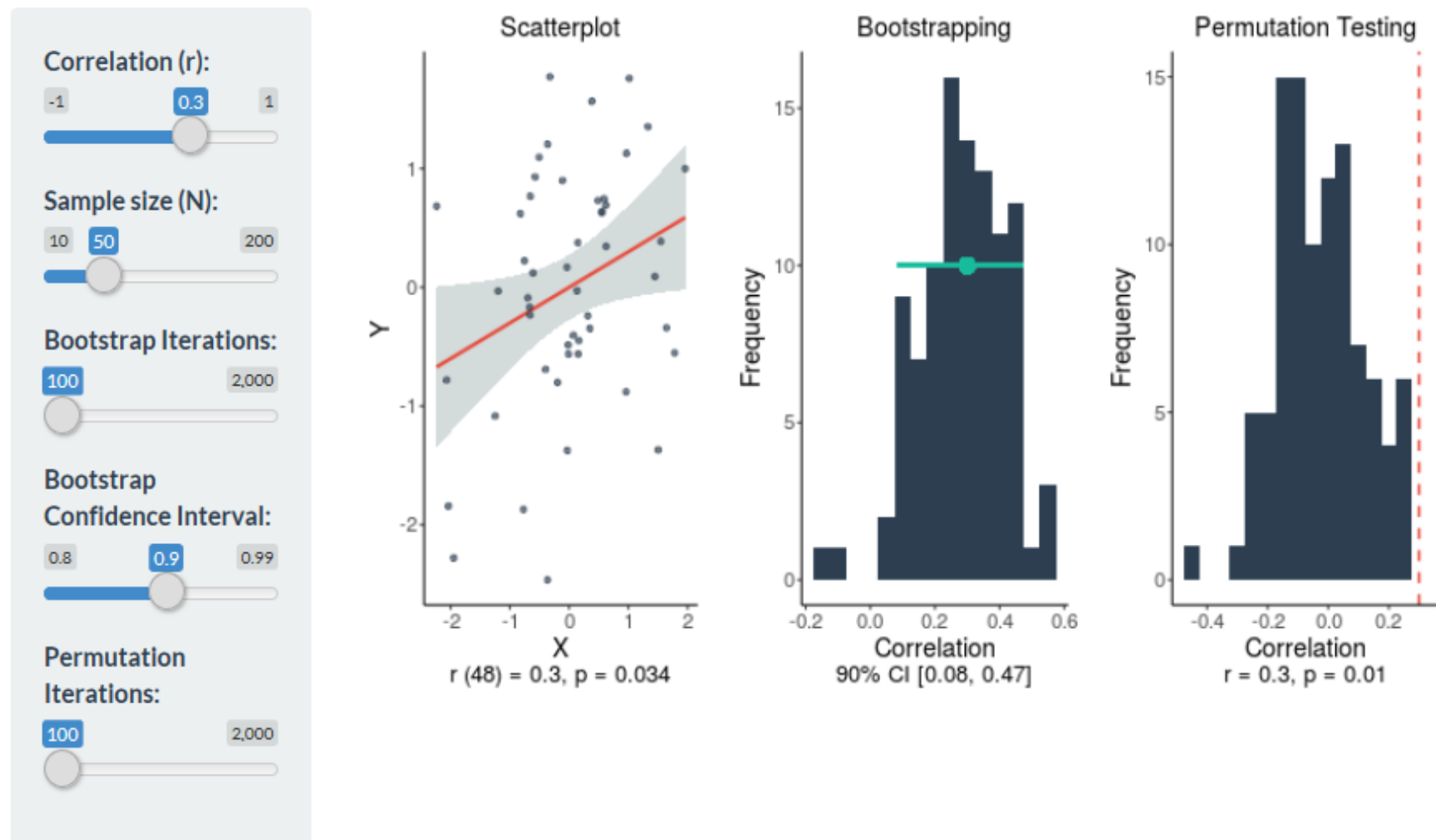
# Basics of bootstrapping



# Bootstrapping and Permutation Testing

Matthew J. Kmiecik & Ekarin E. Pongpipat

See our [blog post](#) for more information about this shiny app.



Source: <https://mattkmiecik.shinyapps.io/boot-perm-app/>

<https://mattkmiecik.shinyapps.io/boot-perm-app/>

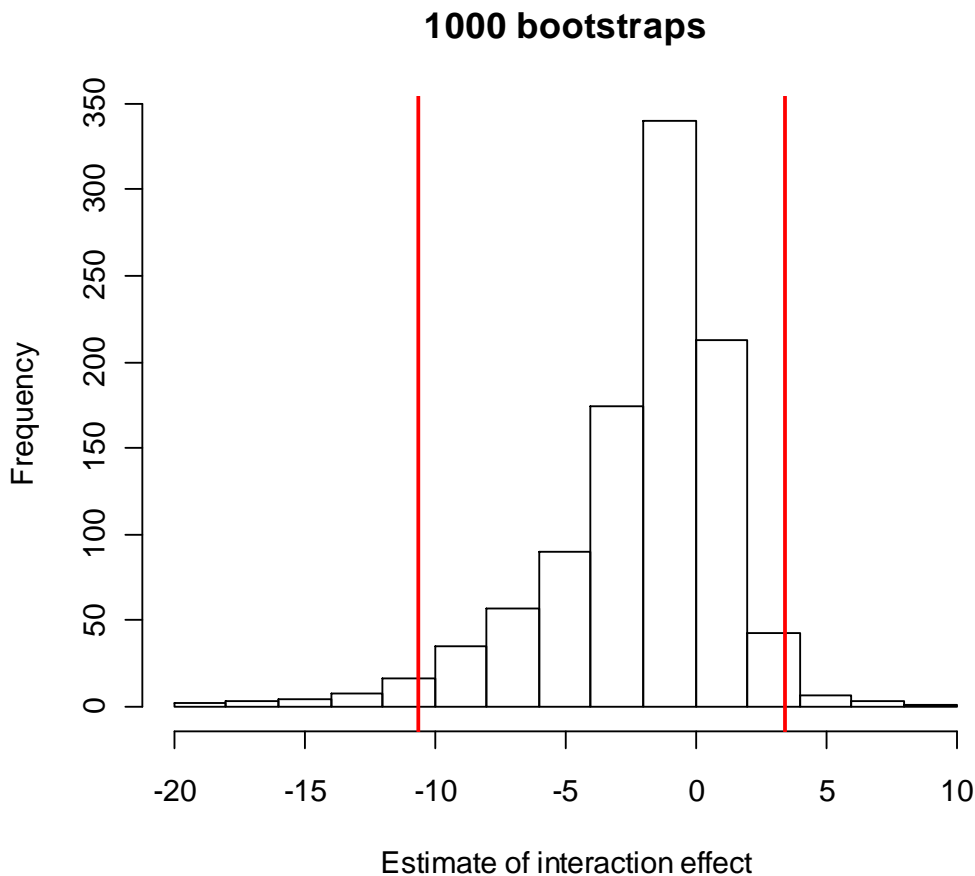
# Solution in lavaan: bootstrapping

1000 bootstraps gives us:

- 1000 estimates of every parameter, including indirect effect
- The mean of these 1000 estimates = the parameter estimate
- The SD of these 1000 estimates = the SE of the parameter
- **The .025 and .975 quantiles of these 1000 estimates = the (non-parametric) 95% confidence interval**

# Bootstrap confidence interval

Bootstrap samples **approximate the sampling distribution**.

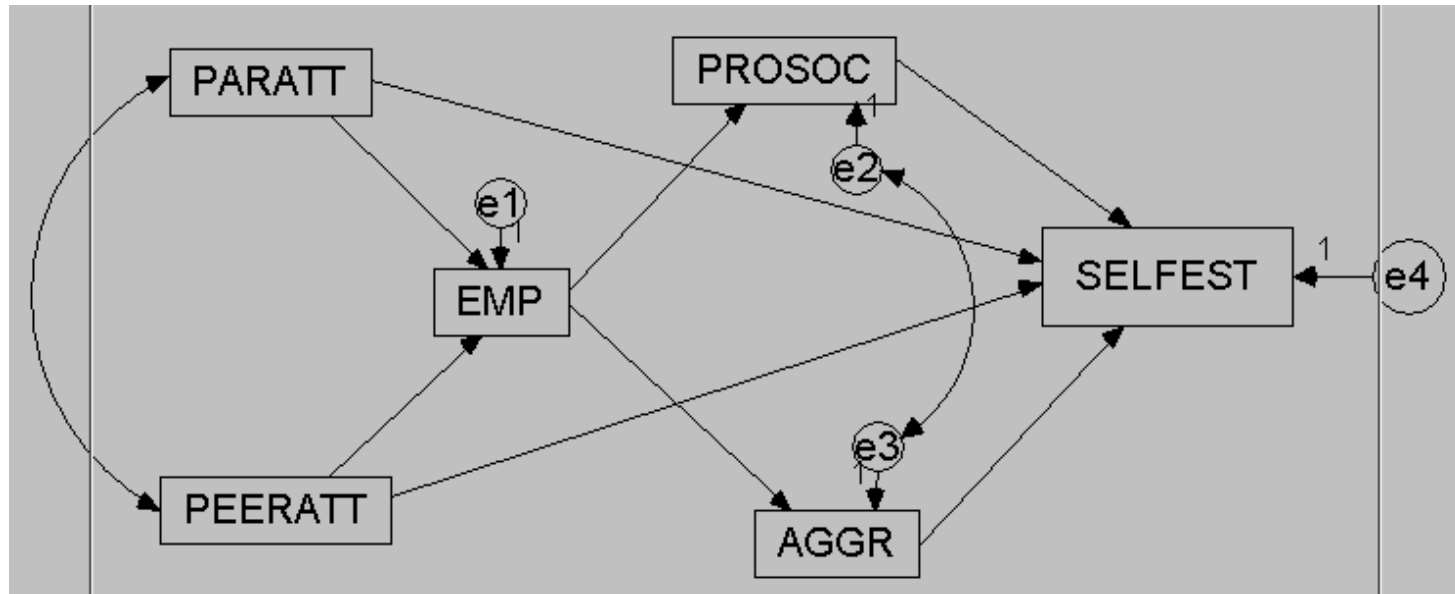


We obtain a lower and upper bound of the 95% confidence interval.

If zero lies inside this interval, we conclude the parameter estimate does not differ significantly from zero.

Hence, there is no indirect effect.

# Model with indirect effects



# Lavaan syntax for indirect effects

```
model <- ' # direct effect
          Y ~ c*X
          # mediator
          M ~ a*X
          Y ~ b*M
          # indirect effect (a*b)
          ab := a*b
          # total effect
          total := c + (a*b)
          '
```

# Lavaan output for indirect effects

## Regressions:

		Estimate	Std.Err	z-value	P(> z )
Y ~					
X	(c)	0.036	0.104	0.348	0.728
M ~					
X	(a)	0.474	0.103	4.613	0.000
Y ~					
M	(b)	0.788	0.092	8.539	0.000

## Variances:

	Estimate	Std.Err	z-value	P(> z )
.Y	0.898	0.127	7.071	0.000
.M	1.054	0.149	7.071	0.000

## Defined Parameters:

	Estimate	Std.Err	z-value	P(> z )
ab	0.374	0.092	4.059	0.000
total	0.410	0.125	3.287	0.001



# Lavaan syntax for bootstrapped SE

```
fit <- sem(mediation_model,  
          data = data,  
          se = "bootstrap",  
          bootstrap = 1000)
```

To obtain the confidence intervals, use the following syntax:

```
parameterestimates(fit, boot.ci.type = "bca.simple")
```

lhs	op	rhs	label	est	se	z	pvalue	ci.lower	ci.upper
Y	~	X	c	0.036	0.116	0.312	0.755	0.105	0.070
M	~	X	a	0.474	0.098	4.837	0.000		
Y	~	M	b	0.788	0.094	8.361	0.000		
Y	~~	Y		0.898	0.149	6.044	0.000		
M	~~	M		1.054	0.178	5.917	0.000		
X	~~	X		0.999	0.000	NA	NA		
ab	:=	a*b	ab	0.374	0.087	4.314	0.000	0.213	0.559
total	:=	c+(a*b)	total	0.410	0.139	2.942	0.003	0.140	0.689

Zero is not in C.I.:  
Indirect effect  
significant

# P-value based on bootstrapping

P-value is based on bootstrapped standard errors if  
you specify `se = "bootstrap"`

# Other use of bootstrapping

Bootstrapping is also useful:

- if **sample size is small**, such that normal approximations are not appropriate
- if the data are (multivariate) **non-normally** distributed

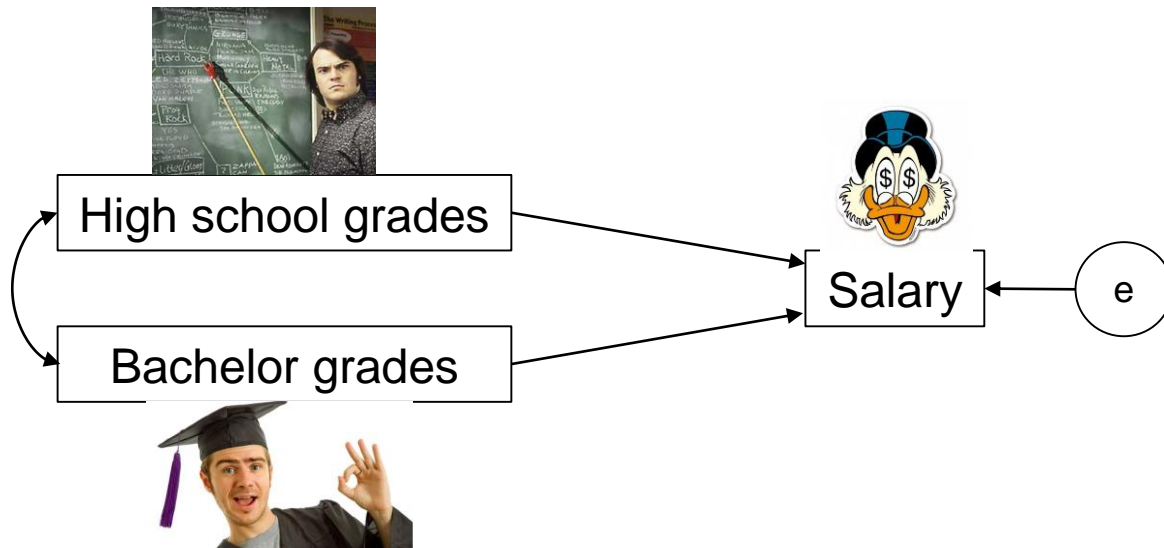
Some people say: Bootstrap everything, all the time.

This allows you to relax the assumption of normality

Different causal models

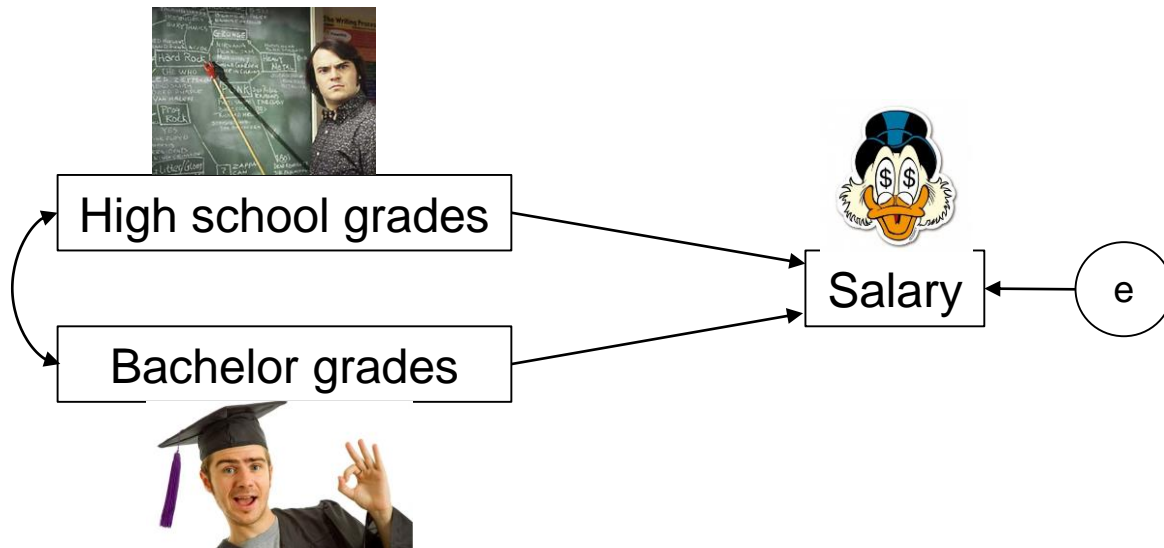
# Causality

- You have several correlated variables
- You're imposing a "causal structure" on the variables
- E.g., these two predictors (IVs) are correlated
- You can ask **why** they are correlated.



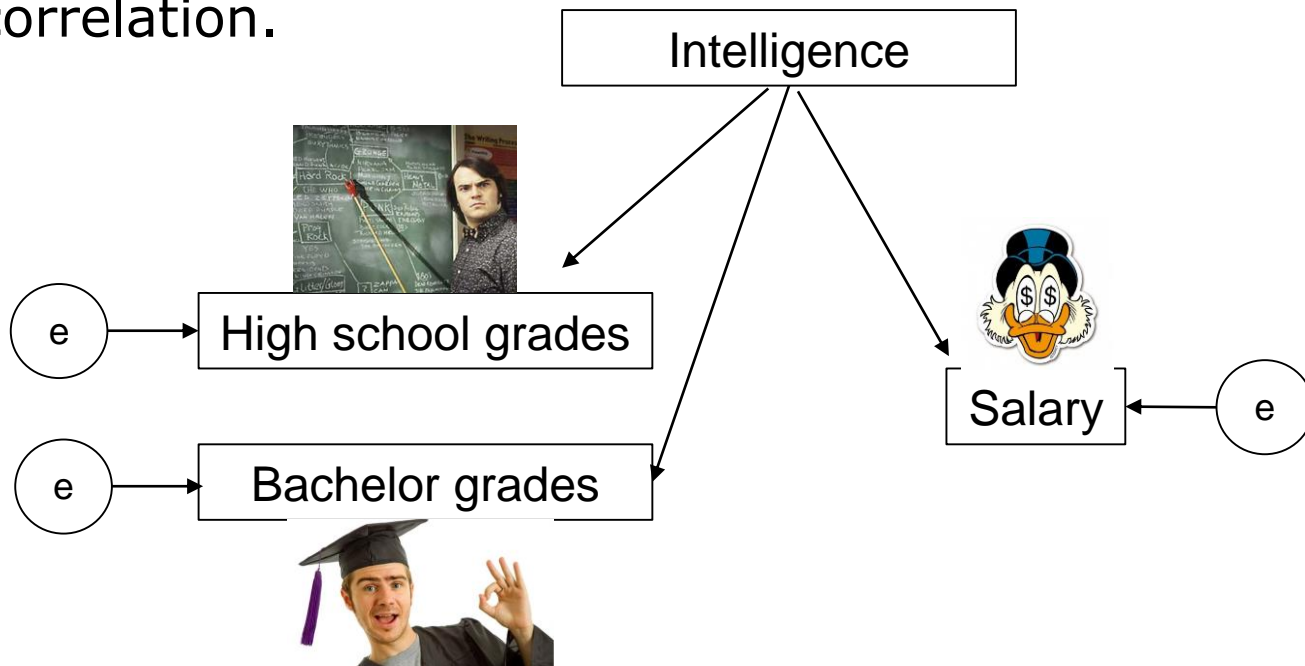
# Causality

- These two predictors (IVs) are correlated
- You may have a theory about **why** they are correlated:
  - $X_1$  may influence  $X_2$  directly (or “reverse causality”)
  - Alternatively: A third variable  $X_3$  is responsible for the correlation.



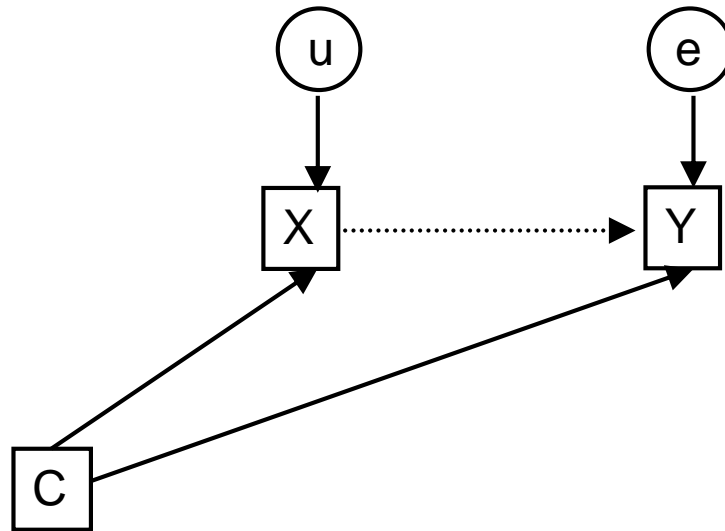
# Causality

- These two predictors (IVs) are correlated
- You may have a theory about **why** they are correlated:
  - $X_1$  may influence  $X_2$  directly (or “reverse causality”)
  - Alternatively: A third variable  $X_3$  is responsible for the correlation.



# Confounders

A confounder is a third variable that once it is included, changes the relationship between X and Y.



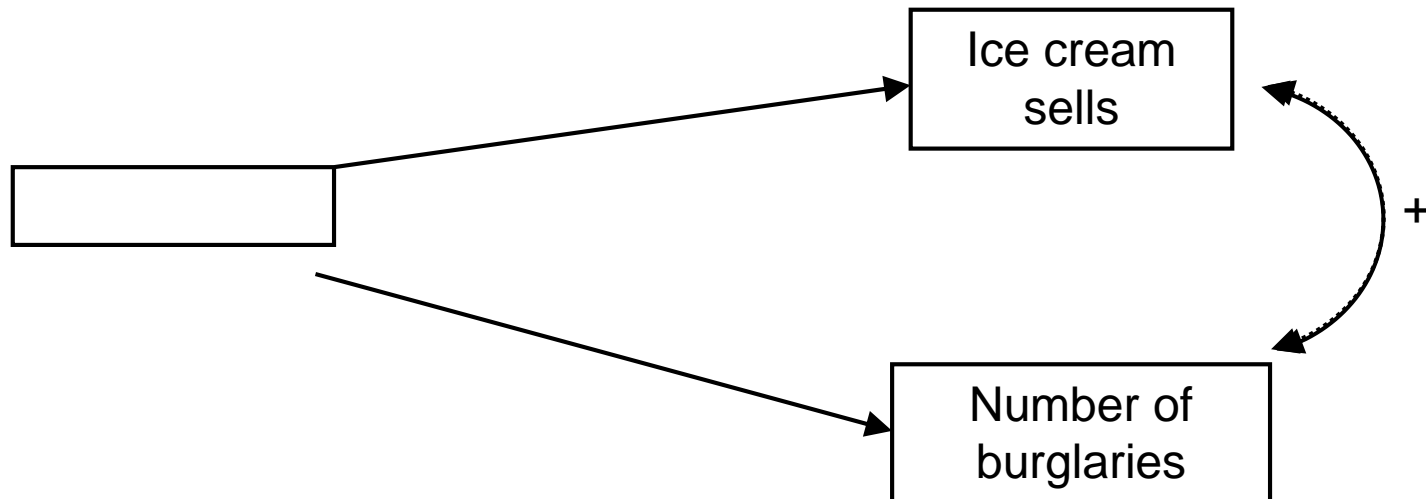


# Spurious effects

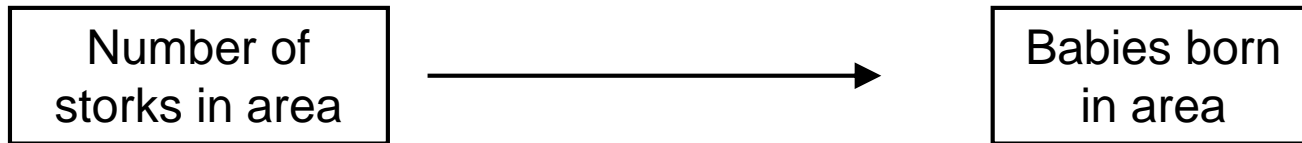
X is associated with Y, because Z causes X and Y.

The relationship between X and Y is spurious.

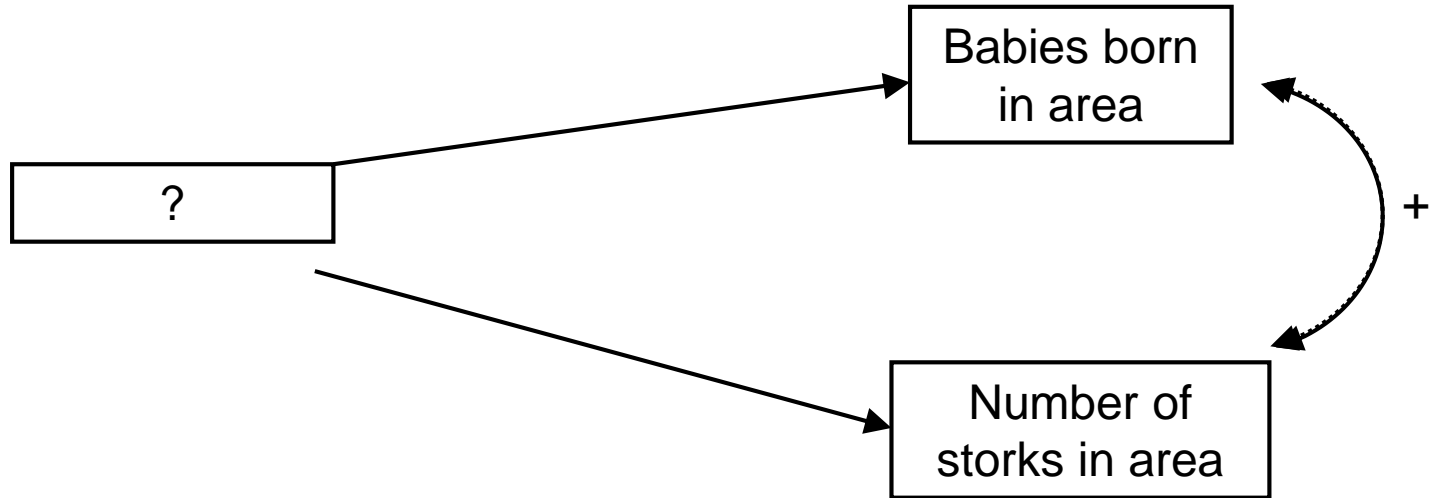
**Warning:** If you analyze these data with mediation model, you will probably find significant mediation. Why?



# Spurious effects?



# Spurious effects?



# Causality

- If I fit a mediation model
- And the model has good fit
- Can I conclude that the effect of X on Y is indeed explained/mediated by M?

# Causality

- If I fit a mediation model
- And the model has good fit
- Can I conclude that the effect of X on Y is indeed explained/mediated by M?

- 
- **NO!** Causality is always in the **METHODS** (or theory), not in the **STATISTICS**
  - My model reflects my theory
  - Different causal models will have identical fit (if you flip some of the paths around)